The Value of Teaching Mathematics

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Introduction

While government and industry keep praising mathematics for it's usefulness and importance in life, it can be hard for children to see how functions, equations or geometric shapes can help them in everyday life. And together with its difficulty, this may be one of the reasons why mathematics is so unpopular amongst many students.

However *practical value* is only one of the reasons why we study mathematics. In the 1996 UNESCO Report *Learning: The Treasures Within* [1], Jacques Delors describes four "Pillars of Education": learning to *know*, learning to *do*, learning to *live together* and learning to *be*. The following similar distinction is also based on the ones by Peter Gill in [2] and Eizo Nagasaki in [3], section 4.3.

- Most obviously, school teaches information of **practical value**: how to do simple arithmetic, grammar, capitals of the world or simple human biology. This broadly corresponds to Delors' *learning to know*.
- But many subjects are not directly useful in everyday life. School also teaches **discipline**: working efficiently, reasoning, teamwork, keeping a schedule – and doing things you don't enjoy (like homework). This broadly corresponds to Delors' *learning to do* and *learning to live together*.
- Finally, many subjects in school have a **cultural value**, such as music and art, history and geography. Science and maths, as knowledge about our universe, may also have an intrinsic cultural value. This broadly corresponds to Delors' *learning to be*.

Mathematics is one of few subjects that has the potential to range across all three of these areas and the table below shows examples of what could be taught in each case.

Practical Value *'Numeracy'*	**Disciplinary Value** *Reasoning and Logic*	**Cultural Value** *'Fundamental' Mathematics*
• Numbers and Calculating • Algebra, Geometry, Functions, Calculus, etc. • Probability, Statistics and Data Analysis	• Finding new solutions to problems of a kind that have not been seen before. • Proof simple theorems using logic.	• Puzzles, Number Theory, Combinatorics, etc. • History of Mathematics • Biographies • Unsolved Problems

Table 1: Aspects of Mathematics Education

The titles of these categories may be somewhat misleading, and there can be significant overlap. For example, *reasoning* clearly has a practical value in life, while topics like calculus are of little practical value outside engineering, scientific research and finance.

For the purpose of this essay, *practical value* will refer to learning particular methods and algorithms to solve certain problems or interpret data. These may or may not be 'useful' in everyday life.

The *disciplinary value* will refer to a *mathematical way of thinking* – problem solving, reasoning, logic and proof – with 'discipline' meaning 'field of study', rather than 'behaviour'. These skills may be taught in mathematics but can be useful in many other subjects.

To make mathematics as engaging, interesting and relevant as possible, the curriculum should be a combination of all three aspects. Unfortunately the current focus of mathematics education, particularly in the UK, is very much the first column of Table 1: memorising methods to solve particular problems, such as solving quadratic equations, and applying them to standard exam question. This doesn't work without simple reasoning, but rarely to the point which one would call *doing mathematics*.

In this essay I will discuss the importance of mathematics and mathematics education, drawing upon a wide range of research and personal experience, and following the three categories above.

In particular, I want to find evidence against the overarching opinion in Bramall and White's '*Why Learn Maths?*' [4], who argue that since the majority of people don't use anything beyond primary mathematics in everyday life, its privileged position of mathematics in schools may be unjustified.

1 The Practical Value of Mathematics

> *Numbers exist only in our minds. There is no physical entity that is number* 1*. If there were,* 1 *would be in a place of honour in some great museum of science, and past it would file a steady stream of mathematicians gazing at* 1 *in wonder and awe.*
>
> FRALEIGH & BEAUREGARD, Linear Algebra

Link to School Mathematics

In this chapter will try to assess the practical value of mathematics. I will draw a distinction between the usefulness of mathematics and the effectiveness of school in preparing children for using mathematics. I will also consider the effect of calculators and computers on these arguments, and give a variety of examples.

Though certainly important to consider, the focus will not be on how mathematics is used practically by engineers and scientists, but on how it is used in *everyday life* by the majority of the population. "Everyday life" can include the workplace, for example hospitals or retail banks, but no jobs which require mathematical or scientific training beyond A-level, such as software engineering or stock market trading.

Mathematics in Life

It is very hard to come up with an area of mathematics which has no application in life.

- Prime numbers, for example, underly digital cryptography and are used whenever an email is sent or a secure website is accessed – from personal internet banking to secret services. This is done using a process called 'RSA cryptography'.
- Fractals are some of the 'least real' objects in geometry, with infinite detail and fractional dimensions. However they can be used in image compression when reducing the file size.
- Vectors are used to define 3-dimensional environments in computer games or engineering software, and multiplied by matrices when rotating or transforming.
- Group theory can be used to study the symmetries of fundamental particles and underlies String theory and other parts of particle physics.
- Logic and set theory is very important in computer science, in particular computability theory.

More advanced areas of mathematics, such as Algebraic Topology or Category Theory, may not have obvious real life applications. However their study can easily be justified since it is impossible to predict how they could be used in the future, and because it is not unusual for links to appear between apparently unrelated areas of mathematics.

There is no doubt that mathematics is of immense practical value in life. However all the examples above only apply to mathematicians, computer scientists and engineers who create these systems – you don't need to know about matrices in oder to play a 3D computer game!

In the third chapter of the essay I will argue that since mathematics is such a crucial part of just about every aspect of our lives it is part of our culture, and we should know what these areas of mathematics are about even if we can't use them. For this chapter, let us return to mathematics used in *everyday* life – as defined above.

Mathematics in *Everyday* Life

We regularly use mathematics in our everyday life: from measuring distances and weights to reading timetables, estimating how much money we spent while shopping and interpreting percentages in newspapers. Many of these skills are taught at primary school level.

In chapter 6 of [3], John White argues that *'the basic arithmetic required for many jobs will be largely acquired by the end of primary school'*. Of course some jobs *do* require more advanced mathematics, but only a minority which – according to John White – does not justify the importance that is given to mathematics in the school curriculum.

A large part of the secondary mathematics curriculum does indeed *not* seem very useful in everyday life: from solving quadratic equations to sketching graphs, long division or trigonometry. The importance of these areas in the curriculum and for exams explains opinions like the one above.

However there are certainly a number of aspects of GCSE mathematics that *can* be useful in life, or at least *more efficient* than relying on intuition. (And many students who study A-level mathematics expect to study technical subjects like science or economics at university.) Here are a few examples:

- Choosing between different mobile phone, TV or internet contracts based on your average usage, when the offers consist of a fixed and a variable price. (Linear functions and equations)
- Understanding and interpreting percentages and graphs in newspapers, such as probability based weather reports, election polls and risk assessments.
- Calculating interest and compound interest, taxes, mortgages and other personal finance. There are countless online tools for this, but you still need to know what the various numbers and results mean.

In addition, having studied GCSE or A-level mathematics may open up many opportunities later in life: for example management positions which require a certain amount of business strategy, performance analysis or financial planning. In fact, according to [5], studying A-level mathematics increases the average expected salary by as much as 10%!

Since we are primarily thinking about school mathematics, it is worth observing that many of the skills taught in mathematics are required in other subjects. This is clearly true for sciences, but also geography, where children may have to find the area of countries or distance between cities, or politics, where children may have to interpret data describing our life. There is significant research regarding these transferable skills and whether pupils actually use them, for example [6], however this is beyond the scope of this essay.

Note that these problems in science or geography are essentially mathematical problems. The transferable skill of more general *mathematical thinking and reasoning* will be discussed in the next chapter.

Mathematics Taught in Schools

Whether we are thinking about simple arithmetic or derivative pricing in finance, the key question is whether the mathematics curriculum in schools prepares children sufficiently for using mathematics in life. The research below shows that unfortunately this isn't always the case.

In [7] and [8], Hoyles, Noss and Pozzi compared mathematical ideas used by paediatric nurses, investment bankers and commercial pilots with how the underlying concepts are taught in school. The result was that the two methods are usually quite different: nurses, for example, calculate drug dosages based on proportional reasoning that is often specific to certain drugs or quantities, rather than the general method taught in schools. They arrive at the correct answer, but they may not understand how their methods work and rely heavily on experience and intuition.

This suggests that schools should maybe teach more "mathematical intuition": being able to estimate answers quickly, notice when answers seem unreasonable and decide how to proceed when encountering an unknown problem. This is only possible if students have real life examples to relate to, such as drug dosages, and their effectiveness of these real life examples will be discussed further below.

Some researchers even suggest that the common practice of blindly following algorithms and procedures to arrive at an answer *diminishes* children's natural mathematical intuition. One such example is given by E. Fischbein in [9], who researched the probabilistic intuition of pre-school children – and discovered that their intuition is much better than one would expect from the ability of GCSE students in probability. In real life, we would rarely *calculate* probabilities and rather use intuition and experience. Maybe there should also be a stronger focus of this in schools.

There have been a number of independent and government commissioned studies of how mathematics teaching in schools meets the requirements of *"higher education, employment and adult life in general"*. One of the most famous examples is the Cockroft report from 1978 [10], from which this quote is taken.

One of their results was that many students did not require formal mathematical knowledge to solve problems that were essentially of mathematical nature. In [11], Paul Downling calls this the **Myth of Reference**. Examples quoted by Downling are that knowledge about angles and tessellations if not required to create pavement patterns, and knowledge about advanced aerodynamics if not required to make an afterburner for jet engines.

A more recent report entitled *Mathematics and Democracy: The Case for Quantitative Literacy* was published in the US by the National Council on Education and the Disciplines (NCED), led by Lynn Arthur Steen [12]. As noted in [13], they too distinguished between quantitative skills (such as measuring) and mathematical tools and language (such as formal algebra).

There is no doubt that a formal mathematical language is needed to solve advanced mathematical problems. But based on the kind of mathematics that children will use in everyday life, it seems that this should not be the *focus* of school mathematics.

Using Mathematics to Model the Real World

One approach to make mathematics learned at school appear more useful is to pose questions in a real world context. Downling presents a number of examples in [11]. He notes how all cases are very unrealistic and implausible, and that the ideal mathematical solution may not be very useful in real life. This leads to his **Myth of Reference** – that mathematics can refer to practices other than itself.

I believe that the problem is not so much that mathematics can't be applied to the real world in a school context, but that questions are created in the wrong way. Usually, teachers and textbook authors starts with a mathematical idea that needs to be taught and invent real world problems around them. The more realistic approach would be to start with a real world problem, think – together with the students – about the kind of mathematics necessary to solve the problem, and then link it with various parts of the curriculum. Many interesting examples of this kind are given in [14].

Use of Calculators and Computers

There is another argument against the practical value of mathematics: computers. Many calculations that had to be done by hand only 20 years ago can now be done easily using computers – and the ability of computers will continue to increase in the future. Do students still need to learn any mathematics [15}?

This is certainly an important point to consider. However while computers may be able to solve arithmetic problems, they can't formulate real life problems in terms of arithmetic or algebra, and they can't interpret what the results mean for the initial problem.

I believe that rather than reducing the amount of mathematics that needs to be learned, computers present a great opportunity for the mathematics curriculum in the future. Student's may not need to learn how to do long division or how to solve quadratic equations. This gives space in the curriculum for much more advanced problems. In the past, these would have been too difficult because of complex computations – today, computers can do the boring and tedious parts of mathematics while students can focus on applications, the underlying principles and *mathematical thinking*, which will be discussed in more detail in the next chapter.

One initiative that tries to use computers to *enhance* mathematics, rather than replace it, is the Computer Based Math Project [16], funded by Wolfram Research, the developers of Mathematica. Using the computational power of Mathematica, students could work on extremely realistic and useful real-life problems, from roller coaster design to historical stock price analysis. According to their website,

> *Students should be able to set up a problem, ask the right questions, turn it into math, specify the calculation, and interpret as well as validate the results. Take the 80% of the time spent doing hand calculation and turn it into time learning concepts and creative skills. Let computers do the calculating.*

With computerbasedmath.org, students learn a very different *kind* of mathematics, which may be much more useful, interesting and exciting. It will be incorporated in the Estonian mathematics curriculum this year and it will be interesting to observe the results.

Using Mathematics to Estimate, Criticise and Interpret

Finally I want to mention briefly that mathematics has an important social role. Basic arithmetic and estimation is necessary in everyday life. Understanding and correctly interpreting data it important if one doesn't want to fall into traps when seeing biased advertisements or reading the newspaper. At a more advanced level, one can criticise mathematical and statistical models, approaches and results. Frankenstein [17] goes further by observing that statistics can be used *"to reveal contradictions under the surface, and initiate social change"*.

These problems are a great example of Delors' *learning to live together*, but unfortunately the constraints of this essay don't allow for further discussion.

Summary

This chapter summarised several different opinions on the practical value of mathematics. Some say that mathematics is very important in life, others say that the majority of children won't actually use much of secondary mathematics throughout their life.

The conclusion might be that the mathematics taught in schools is the wrong *kind* of mathematics, focussing too much on memorising algorithms and doing computations. One approach might be to shift focus towards mathematical intuition and mathematical thinking, and will be discussed in the next chapter. Another approach might be to shift focus towards mathematical computing, allowing for much more realistic problems.

Finally it may be worth noting that many of the jobs that advance technology and society as a whole – from computer programming to electrical and mechanical engineering, scientific research, company management or finance – require a significant amount of mathematics. Thus it is clear why government and industry are very concerned by the fact that the number of students with an "advanced understanding" of mathematics is declining [18]. Even those who believe that secondary school mathematics may have little practical value to an individual must agree that teaching mathematics has been of enormous practical value to society as a whole.

2 The Disciplinary Value of Mathematics

> *A mathematician, like a painter or poet, is a maker of patterns. If his patterns are more permanent than theirs, it is because they are made with ideas.*
>
> G. H. Hardy

Overview

The previous chapter focussed on learning mathematics in order to solve mathematical problems: whether they are posed in an abstract context or applied to the real world (including science or geography lessons at school, engineering or personal finance).

This chapter will focus on *mathematical thinking and reasoning* an independent skill, which can be applied to mathematical problems but also a wide range of other problems, school subjects, or aspects of life.

What is "Mathematical Thinking"?

Traditional mathematics teaching, common in both UK and US, consists mostly of memorising facts and formulas, applying certain procedures and algorithms, and – to a lesser extent – understanding some of the underlying concepts [19, and many similar]. However it rarely happens that students have to *do* mathematics, for example by exploring unknown problems.

In *A Mathematician's Lament* [20], Paul Lockhart compares this to music students learning to read and transpose notes without ever hearing or playing any music. Or artists learning about colours, brushes and Paint-by-Numbers without ever painting anything new.

I believe that it should be at least as important to learn to 'think mathematically', as it is to learn arithmetic and algebra. However it is quite hard to define *mathematical thinking*, and this is summarised by Seymour Papert in *Teaching Children to be Mathematicians vs. Teaching about Mathematics* [21]:

> *In becoming a mathematician does one learn something other and more general than the specific content of particular mathematical topics? Is there such a thing as a* Mathematical Way of Thinking*? Can this be learned and taught? Once one has acquired it, does it then become quite easy to learn* particular *topics – like the ones that obsess our elitist and practical critics?*

Papert refers to *elitists* as mathematicians believing that "trivia" taught in schools disturbs the study of "true mathematical structures" while *practical critics* think that only real life arithmetic skills need be practiced.

In the following part of his essay, Papert gives an example of such a *Mathematical Way of Thinking*, which is verbalised in the computer language LOGO. He describes how playing with LOGO will lead to a mathematical understanding that then makes it much easier to learn topics like algebra or geometry.

While computer programming is only one vehicle that could lead to such a *Mathematical Way of Thinking*, all approaches must all have several features in common:

- Children have to be able to explore, investigate and play, with a project oriented approach (rather than problem oriented). This also includes *debugging*: finding errors or mistakes in their working.
- It is helpful to have a physical object to think about, for example a model plane controlled by a computer program. There is also a need for a precise language to think about and communicate the resulting ideas.
- Children should be able to ‘fall in love’ with the concepts and objects involved.

Papert concludes by noting that “the choice of content material, especially for the early years, should be made primarily as a function of it’s suitability for developing [these non-formal mathematical primitives]”.

Mathematics developing Brainpower and Training the Mind

Mathematics is notorious for being hard. While in Geography or History you can always write essays on topics you don’t really understand – even if they are of low quality – Mathematics usually requires a clear grasp of the underlying concepts or necessary results. If you don’t understand a question, it will be almost impossible to attempt a solution.

On the other hand, Mathematics can show what our mind is capable of doing. It is no coincidence that intelligence tests always include many mathematical and logical puzzles. Studying mathematics exercises our brain in a manner that is quite different from most other human activities.

There is much research regarding the relationship between mathematics and the brain, ranging from education to neuroscience. One particularly interesting example was given by Blaira, Gamsonb, Thornec and Bakerd in 2004. In [22] they propose that the significant increase of mean IQ in the United States during the last century (about 20 points!) could be caused by, or at least be related to the increased *“cognitive demand of mathematics curricula for young students”*. Continuing research in this subject, from a variety of disciplines, will provide great insight regarding the power and usefulness of mathematics.

The Value of Logic and Proof

A proof is a collection of logical arguments that establish a result beyond all doubt. In pure mathematics, the proof is often just as important as the final theorem, since it may include many details about *why* it is true and how it relates to other areas of mathematics. Proof is only one part of the mathematical thought process – and usually the very last one. But it is what characterises mathematics, and what makes it different from all other sciences: proof allows us to be *absolutely certain* that certain results are true. [See 23 for details.]

As Gila Hanna notes in [24], the value of proof in some areas and applications of mathematics has recently become more debatable, particularly after the acceptance of computers for proofs (e.g. four colour theorem) or the development of “zero-knowledge proofs” and “holographic proof”.

Proofs themselves may not have a particularly high value in secondary mathematics education – but the process of *developing* a proof certainly does. Asking students to prove a statement forces them to think logically, to examine every statement rigorously, and to justify their explanations. According to Hanna in [24], this is a great opportunity for mathematics teaching, but involves the challenging task of teaching students the rules of mathematical argumentation:

> *We know all too well that many students have difficulty following any sort of logical argument, much less a mathematical proof. […] We need to find ways […] to help students master the skills and gain the understanding they need. Our failure to do so will deny us a valuable teaching tool and deny our students access to a crucial element of mathematics.*

The Precise Language of Mathematics

When writing about mathematics, or when proving a theorem as suggested in the previous section, it is important to have a clear and precise language and to present ideas in a logical and well-structured way. One has to consider all different possibilities and all data or variables required.

Based on my own experience, I believe that this skill is very useful in many other areas of life: from writing essays and journalism to giving talks presentations. In all these cases, information has to be presented in a clear and accessible way, and this works best when using a logical and *mathematical* approach.

Of course there are many other skills involved, such as creativity or language, but a logical structure and concise explanations are certainly a very good start.

Summary

This chapter discussed mathematical thinking as a skill independent of its practical applications in the first chapter. While it can be hard to *define* mathematical thinking, it was easy to find a media to *teach* logical and mathematical thinking: from computer programming in LOGO to teaching proof. However, as noted above, this also involves many challenges.

Throughout the chapter, a number of *values* of mathematical thinking were uncovered. They include increased brainpower and IQ as well as developing a more precise language to express thoughts and ideas.

Highly popular puzzles like Sudoku or the Rubik's Cube show that practicing mathematical and logical reasoning can be far from dry and boring. It would be fantastic if similar excitement could be brought into school mathematics.

3 The Cultural Value of Mathematics

> *Mathematics, rightly viewed, possesses not only truth, but supreme beauty – a beauty cold and austere, like that of sculpture, without appeal to any part of our weaker nature, without the gorgeous trappings of painting or music, yet sublimely pure, and capable of a stern perfection such as only the greatest art can show.*
>
> Bertrand Russell, Study of Mathematics

Overview

This chapter is about the cultural value of teaching topics in mathematics which may not have any practical application in life, at least for the vast majority of students. It is important to note that this is not the same as Ethnomathematics. While Ethnomathematics is about how culture can influence mathematics and mathematics teaching (see [25]), this chapter is about how mathematics adds value to our culture.

Note that because of the nature of this topic, the opinions and examples in this chapter are very subjective and personal. I will try to outline a number of different aspects but, the constraints of this essay don't allow for a detailed discussion.

Mathematics as the Language of the Universe

To start this chapter, I want to quote a leader from the Daily Telegraph (19 August 1998) which appears in [4].

> *Mathematics is the master key to the Universe. Its mysteries may seem arcane to the laymen, but without it we would still be living in a pre-scientific, pre-industrial world. Most educated people have little grasp of the arithmetic and geometry of the Ancient Greeks, yet none of their legacies has a more direct impact on our lives than those of Euclid, Archimedes or Diaphantos. As for the modern immortals, the great discoveries in physics of a Newton or an Einstein would have been impossible had they not also been superb mathematicians. Our society is shamefully ill-equipped to comprehend the mathematical mind.*

There are two legacies of the Ancient Greeks: Democracy and Mathematics. (And it is worth noting that democracy wouldn't work without mathematics to calculate taxes or determine the seat distribution in parliament based on direct and indirect votes.) Without mathematics, there would be no skyscrapers, no television, no computers, no commercial airlines, no spaceflight and no weather forecast. Without mathematics we would not be much more advanced than the ancient Babylonians. The cultural value and the monetary economic value of mathematics are too large to measure.

More importantly, the laws of nature are written in the language of mathematics: from the equations of general relativity that govern the motion of planets, starts and galaxies everywhere in the universe, to the electrochemical signals in our brain. As one drills down further into matter, the underlying mathematics becomes more and more obvious, culminating in Quantum Mechanics.

It is one of humanities most noble endeavours to understand the universe we live in, and that would not be possible without mathematics.

Mathematics as the Language of Technology

The last 50 years have, in an unprecedented way, showcased the power of mathematics: through computers. Today there is very little in life that would work without computers, from barcode scanning when shopping to controlling elevators in skyscrapers.

Learning about Prime Numbers and RSA encryption may give us confidence regarding internet security, while learning about computer programming or network protocols may give us an idea why a computer crashed and how to avoid that.

While all these applications may not have direct practical applications in everyday life, I personally take great satisfaction in understanding something that is so fundamental to my life.

The Beauty of Mathematics

Pure mathematicians are known for passionately talking about the 'beauty' of mathematics – much like a painting or a piece of music. Interesting discussions of this topic are given in Hardy's *'A Mathematician's Apology'* [26] and chapter 9 of Bramall and White's *'Why Learn Maths?'* [4].

It is hard to argue with these arguments: just as you may or may not like classical music you may or may not appreciate the beauty of a particular proof or theorem. But just like any other form of art, mathematics deserves to be respected even by those who can't appreciate it.

Summary

There is no doubt regarding the importance of mathematics in technology and science. However this does not mean that there is any value in children learning about it. The 'beauty' aspect of mathematics is even more controversial.

I personally believe that the great importance of mathematics in our world is reason enough for learning *about* it, even if you can't *do* advanced mathematics – just like you know about Mozart even if you can't play his music.

In [27], Christer Kiselman mentions another cultural value of mathematics: **internationality**. While the symbols and words may be different, mathematical concepts are the same everywhere in the world – and even in alien civilisations in other galaxies. Pure Mathematics truly spans borders and allows for a great amount of international collaboration.

Summary and Conclusion

This essay analysed the value of mathematics from a variety of perspectives: practical value in everyday life, the value of mathematical reasoning, and the cultural value of mathematics. Of course it is impossible to measure these values *quantitatively*, and their relative importance depends very much on the readers personal opinion and experience.

There is no doubt that mathematics as a subject is invaluable, but many doubts have been raised regarding the value of the school mathematics curriculum for the majority of children.

My conclusion is that we shouldn't teach *less* mathematics but *different* mathematics, focussing more on problem solving and 'mathematical thinking', or on mathematical intuition and real life situations, and focussing less on memorising formulas and simply applying algorithms.

Throughout the essay I gave examples of opportunities how the mathematics curriculum could be made more exciting, useful or modern. Presently there are many government and non-government initiatives to change school mathematics. It is a great time to work in education and to observe what will happen during the next years.

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