Polypad Teacher Guide

Polypad is a collection of interactive digital manipulatives designed to be used by both teachers and students. Physical (or concrete) manipulatives have long been a feature in many mathematics classrooms, and their benefits in helping students to develop conceptual understanding are well researched and understood. Digital (or virtual) manipulatives are a comparatively recent addition to the mathematics teacher's toolkit, but research has already shown there are clear advantages to using them in the classroom (Hunt et al, 2011).

Availability is key: not every classroom has access to class sets of many types of physical manipulative, whereas digital manipulatives are available with any computer, tablet or smartphone. Polypad provides a set of digital manipulatives that can be used instead of or alongside physical manipulatives to support students' development of key mathematical concepts in shape, number and algebra.

Mathigon runs on all recent browsers and operating system, and supports touch-screens (including on small smartphone screens). Within our native mobile app for iOS and Android, it also runs completely offline.

UI elements

Sidebar

The sidebar contains all the available shapes and tiles. You can click them to add a copy to our canvas, or you can drag them directly from the sidebar onto the canvas. On small screens, you may have to press the ☰ icon in the bottom left corner to reveal the sidebar.

At the bottom of the sidebar, there are a number of additional tool icons:

🔖	**Save and share:** one of the most useful features for teachers is the ability to save the canvas at any time and share a link with other people. You can also ask students to save their canvases. If they are part of your Mathigon class, they will automatically appear on your teacher dashboard.
☁	**Download image:** You can download your current canvas as a PNG image. *(On mobile devices, we will open a new tab with the image. Long-press on the image and then share/save it as necessary.)*
▦	**Grid options:** change the grid settings
🌐	**Change language:** Polypad is available in 19 different languages.
⟲	**Clear Canvas:** this removes everything from your canvas so you can start again with a blank screen.
📔	**Instructions:** show instructions for use, including keyboard accessibility shortcuts and lesson plans.

Toolbar

The bar at the bottom of the canvas shows all available Polypad tools:

- **Move tool:** select on and move any element on the canvas. You can select multiple elements at once by clicking and dragging around the elements you would like to select.

- **Text tool:** click/tap anywhere on the canvas to create a text box. You can add labels, or encourage students to annotate and explain their work. Later, you can double-click a text box to change its content.

- **Pen tool:** draw freehand on the canvas.

- **Eraser tool:** click and drag to remove elements (tiles or pen strokes) on the canvas.

- **Undo:** revert the last action.

- **Colour picker:** change the colour of the currently selected tiles, or of the pen. You can choose from a preset of colours, or use RGB sliders to create custom colours.

If you select one or more tiles on the canvas, additional **dynamic actions** will appear floating above the toolbar. There is always a ⊗ delete icon, and a ⊞ copy icon, to duplicate the current selection.

Depending on which type of tile you have selected, additional action buttons may be available. For example, a "flip" action will appear when selecting polygons, and a "split or merge tiles" action will appear when selecting number tiles.

Selecting and moving tiles

To move a tile around the canvas, simply click/tap and hold while dragging it. Tiles will snap to each others' vertices, or the grid points if you have selected a background grid.

You can select a single tile by clicking on it, or multiple tiles by dragging a rectangular selection box across the canvas. When you have a keyboard, you can also hold the SHIFT key and then click multiple tiles you want to select at once.

Selected tiles have a solid black outline and a small handle that can be used to rotate them. The angle of rotation will automatically snap to certain values that would line up the edges of your current selection with the edges of nearby tiles. This is particularly useful when creating tessellations.

Tile types

Polygons (pattern blocks)

Polypad includes eight **regular polygons**: equilateral triangle, square, pentagon, hexagon, heptagon, octagon, decagon and dodecagon. They all have the same side length so they are ideal for exploring tessellations as well as angle properties.

It also includes a carefully chosen selection of **irregular polygons**, including "21st-century pattern blocks":

The green irregular hexagon (chevron), trapezium and blue rhombus all have angles of 60° and 120°. The red rhombus has angles of 30° and 150°. There is a 2 by 1 rectangle, an isosceles right-angled triangle, and a 30°/60°/90° triangle. The kite has angles of 60°, 90° and 120°, and the arrowhead has angles of 30°, 60° and 240°.

Number Tiles

You can add **number tiles** to the canvas in hundreds, tens or ones. Use the "duplicate" action to quickly create representations for any integers. For example, this image shows the number 132.

Every block consisting of more than one number tile also shows has a black handle when selected. Dragging this handle left and right allows you to change the number of columns in the block. This can be useful for exploring factors and remainders. For example, 24 can be displayed as two rows of 10 and one row of 4, or four rows of 6.
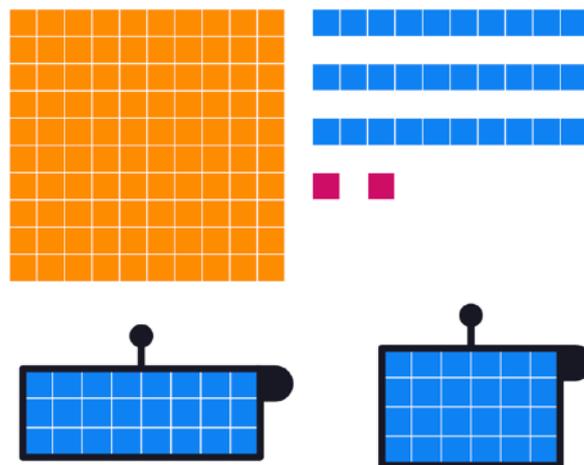
If you select multiple number tiles on the canvas, you can join them into a single block using the "merge tiles" action at the bottom of the screen. Similarly, you can split a block back into individual tiles using the "split tiles" action.

Number Bars (Cuisenaire Rods)

Polypad includes a set of **number bars** from 1 to 10. These are great for comparing numbers, exploring number bonds, and could be used together with the number tiles to explore numbers larger than 10.
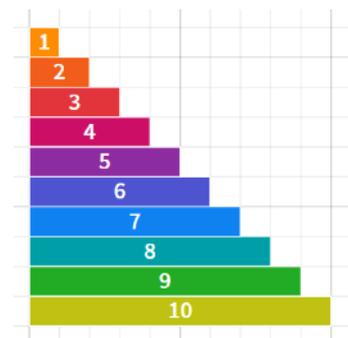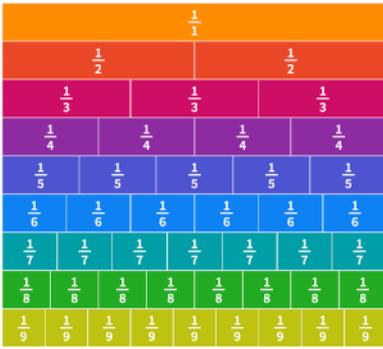
Fraction Bars

Polypad includes a set of **fraction bars** with unit fractions from 1/1 to 1/9. They can be used to make a fraction wall like the one shown here, to help students develop an understanding of equivalent fractions.

Just like number tiles, every fraction bar has a handle to increase or decrease the number of fractions in a row. You can also split a row of fraction bars into individual tiles using the "split tiles" action.

Fraction Circles

Similarly, there are fraction circles from 1 to 1/9, which can be rotated and easily joined together to visualise different proportions.
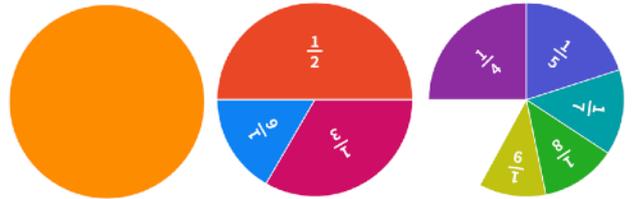
Geometry

This section is slightly different from the other tool types available. Simply click the **ruler** or **protractor** symbol to toggle the corresponding tool on the canvas. You can drag the tools around the canvas just like other tiles, or use the black handles for precise adjustments (including snapping to nearby shapes).
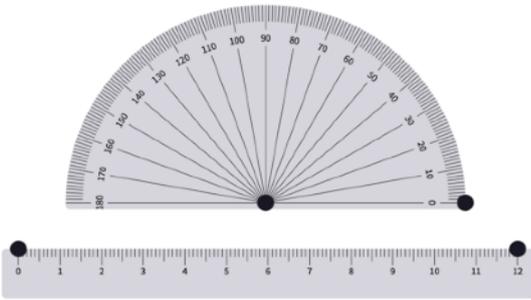
With the *pen tool* selected, you can create straight lines or perfect circle arcs by drawing close to the edge of the ruler or protractor.

Algebra Tiles

The **algebra tiles** included in Polypad offer a powerful visual representation for teaching algebra. There are six tiles available: x^2, x, **1** and $-x^2$, $-x$, **−1**. The diagram on the right shows how algebra tiles can be used to factor

$$x^2 + 3x + 2 = (x + 2)(x + 1)$$

You can use the **grid axis** to show multiplication problems, and use the handles at the ends of the horizontal and vertical bars to resize the grid.
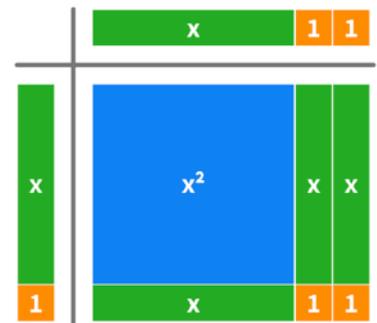
When you overlap a positive and a negative tile, both tiles are greyed out:
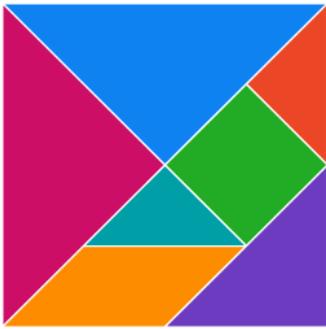
Tangram

Tangram is a traditional dissection puzzle consisting of 7 pieces which can be cut from a square: two large right-angled triangles, 1 medium right-angled triangle, 2 small right-angled triangles, a square and a parallelogram.

Pieces can be arranged to make a wide variety of shapes. You can find many puzzles and examples at mathigon.org/tangram. The tangram pieces also have interesting geometrical properties to explore.

Pentominoes

The **pentominoes**, shown here on a square dotty canvas, are the twelve shapes that can be made by joining 5 equal-sized squares edge-to-edge. (A nice classroom activity is to invite students to find the full set, without telling them how many there are!)
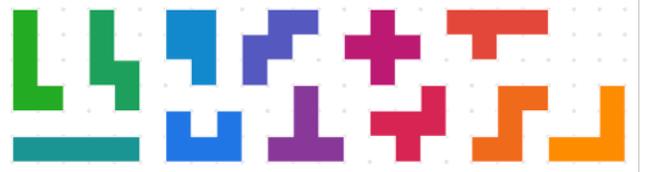
Tetrominoes

The **tetrominoes**, shown here on a square dotted canvas, are the five shapes that can be made by joining 4 equal-sized squares edge-to-edge. Students may recognise these if they are familiar with the game Tetris; the seven Tetris pieces are these five tetrominoes together with two mirror images.

Penrose Tiles

A **Penrose Tiling** is a special kind of tessellation that is aperiodic: this means that whichever way you combine the tiles, the resulting pattern will never repeat horizontally or vertically. Our Penrose tiles consist of two different kites, but there are many other options. When connecting them, the only rule is that the white circle arcs drawn on the tiles always have to line up when connecting adjacent tiles.

Sharing and Teacher Dashboards

If you are signed in to your Mathigon account, you can save any Polypad state by opening the 🔖 **Save and Share** panel in the sidebar footer, and then clicking ➕ **Save current canvas**.

Every saved Polypad has a unique URL which you can share with others. This can help you save time in lessons by preparing an initial canvas in advance. Students can start work immediately without having to set up the manipulatives you want them to use. *Please note, once you have saved a Polypad, the link will not reflect any new changes. If you want to edit a Polypad, just save it again and share the new link.*

Students can save and share their Polypads in the same way. If you have a teacher account, you can create a new class at mathigon.org/dashboard. Every class has a unique 8-digit code, which you can use to invite students. You can also import an entire class roster from Google Classroom. In the sidebar of every class dashboard is a link to a page that shows all saved canvases of students in your class.

You can also specify which sidebar panel should be open using the #hash of the URL. For example, mathigon.org/polypad#algebra-tiles opens the algebra tiles in the sidebar.

Lesson Plans

We have many activities and lesson plans available for Polypad. Take a look at
mathigon.org/polypad/lessons.

Keyboard Shortcuts

Polypad supports many different keyboard shortcuts for accessibility and ease-of-use:

SHIFT	Hold and click to select multiple tiles at once,
CTRL + X	Undo *(Use CMD on macOS)*
CTRL + Y	Redo *(Use CMD on macOS. Alternatively, use CTL + SHIFT + X)*
ARROWS	Move the currently selected tiles around the canvas.
BACKSPACE	Delete the currently selected tiles.
ESC	Deselect all tiles, and close any open popups and modals.
C	Duplicate the currently selected tiles.
R	Rotate the currently selected tile by 15°. Press shift to rotate by –15°.
S	Move the focus to the beginning of the sidebar.
T	Move the focus to the beginning of the toolbar.
U	Move the focus to the canvas area.