Mathigon – Lesson Plan

How Many Rectangles?

Standards

NC Year 5: Identify multiples and factors, including finding all factor pairs of a number

Key Stage 3: Use the concepts and vocabulary of prime numbers, factors (or divisors), multiples

CCSS 4.OA.B.4: Find all factor pairs for a whole number in the range 1–100.

This lesson is designed for upper elementary or lower middle school but can be used with grades 3 and up.

Prior knowledge

It is useful for students to be familiar with multiplication tables – Mathigon's Multiplication by Heart is a great intervention for students who are not yet fluent. Students may find it useful to use the vocabulary *factor, multiple* and *prime;* the warm-up activity could be used to review these terms.

Lesson outline

This task uses Polypad's **number tiles** to provide a clear representation that helps students develop a better understanding of factors, multiples and prime numbers. The dynamic visual representations of number tiles creating rectangles give students a model for factor pairs of whole numbers and for prime numbers. This representation also shows division with remainders: the number of columns shows the divisor and the left over blocks in the last row shows the remainder. It can even be used for some enrichment work on elementary number theory, such as modular arithmetic.

Lesson objective: Understand how to find all the factors of a number.

Lesson activity

Warm-up

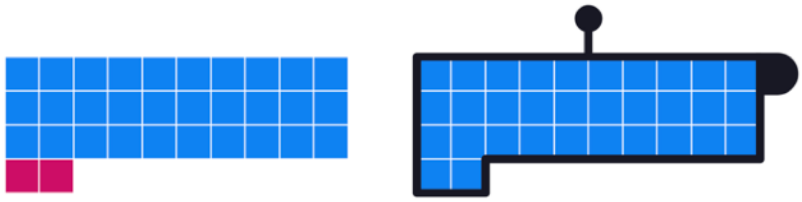
Invite everyone in the class to choose a whole number between 1 and 100, write it down, and then hold it up for everyone to see. Then ask them to look around the classroom – can they see someone who chose a number which is a *factor* of their number? Can they see someone who chose a number which is a *multiple* of their number? Take some time to discuss their findings, perhaps using the following prompts for discussion:

- Did anyone find it really easy to find multiples of your number? What number did you choose?
- Did anyone find it hard to find factors of your number? What number did you choose?
- If someone in the class chose 1: Which number could everyone connect to?

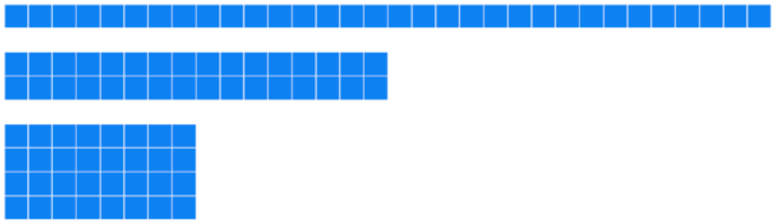
This Polypad shows different division problems that can be represented using number tiles. Show students the blue examples, and ask them to write down the division problem, before revealing the green and red solution.

Main Activity

Create the number 32 using **three tens** and **two ones** (left) and show students how to select all tiles and use the "merge tiles" button to create a single block of 32 (right):



Next, demonstrate how dragging the black handle on the right changes the number of columns. Invite students to predict which arrangements of the 32 blocks will create a complete rectangle, then test to see whether they are right. Class discussion should result in an agreement that there are six possible rectangles that can be made using the number 32: 1 by 32, 2 by 16, 4 by 8, 8 by 4, 16 by 2, and 32 by 1. *Clarify that for this activity, we are considering 1 by 32 and 32 by 1 as two separate rectangles even though they are rotations of each other.*



Now give students some time to explore the question *"How many rectangles?"* with numbers of their choosing. Remind them of the importance of recording their results. You could model how to record all the rectangles for a particular number using 32 as an example.

[This Polypad] shows an example for the number 12. Students can share their Polypad through Mathigon's teacher dashboard, or send you a link to their Polypad through your LMS.

To start off with, you should suggest that students choose a number smaller than 30 – otherwise their widest rectangle may not fit on the screen. It might also be necessary to discuss the importance of working systematically, by starting from an extreme rectangle and moving one step at a time, to be sure not missing any rectangles.

After students have tried a few examples, invite them to share any mathematical questions that might have occurred to them. Here are a few suggested questions that might be suggested, or that you could model if the class is unfamiliar with posing their own questions:

- Which numbers can be made into the greatest number of rectangles?
- Which numbers can only be made into two rectangles?
- Are there any numbers which can be made into an odd number of rectangles?
- Is there a way to work out how many rectangles a number can be made into, without having to make them all?

Once some fruitful questions have been posed, give students some more time to explore the answers to their questions, perhaps working with a partner or in a small group. Towards the end of the lesson bring the class back together to resolve the answers to the questions – students could be invited to present their solution to a question to the rest of the class, explaining how they reached their conclusions.

Possible solutions

The numbers which can be made into the greatest number of rectangles are those with the greatest number of factors. Both 24 and 30 have 8 factors, which is the most for numbers up to 30. If students explore numbers up to 100, they may discover several numbers with 12 factors. Consider challenging students to find all of them: 60, 72, 84, 90, and 96.

The numbers which can only be made into two rectangles are the **prime numbers**. This can be a useful mental model for students who have difficulty remembering the definition of prime.

An odd number of rectangles comes from a number with a repeated factor (that is a square number). Each of the rectangles can be drawn two ways round, but the square just adds one new rectangle to the total. Students might spot this from their work with numbers under 30 by noticing that 1, 4, 9, 16 and 25 have 1, 3, 3, 5 and 3 factors respectively. It is worth taking the time to discuss why square numbers have an odd number of factors, as this helps students make the connection between different properties of numbers, and will help them when they come to work on prime factorisation.

Determining how many factors a number has *without* actually trying all possible rectangle sizes could develop into an entire lesson on its own – see some suggestions below.

Support and extension

For students who are struggling, suggest that they work systematically by starting with 1 and recording the number of rectangles for each number in turn. It might be useful for students to work with a partner where one uses Polypad and the other records the findings, swapping over after each activity.

The question *"How can I find the number of factors that a number has?"* is a rich question for further exploration. It could be used to introduce **factor trees** to find the unique prime factorisation of a number, and Mathigon's prime numbers course has some more content about this. Students can then consider how prime factors raised to different powers contribute to each of the factors of a number. For example, you could challenge students to find numbers that have a specific number of factors (like 8), and then look at their prime factorisation.

Students might notice that some numbers have many factors while others have very few. Ask students to explore adding up the factors of a number (other than the number itself) and see what patterns they notice. This introductory question leads to an exploration of abundant, deficient, and perfect numbers, and you can read more about these in Mathigon's course about sequences.